MULTIMEDIA UNIVERSITY

STUDENT IDENTIFICATION NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2021/2022

BEM1014 – MATHEMATICS

(All Sections / Groups)

05 OCTOBER 2022
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 4 pages, including a list of formulae.
2. Attempt all 4 questions. The distribution of marks for each question is given.
3. Students are allowed to use scientific calculators.
4. Please write your answers in the **Answer Booklet** provided.

Question 1 (25 marks)

a) Find an equation of the line passing through the point (-3,7) and perpendicular to the line passing through the points (2,8) and (4,12). [5 marks]

b) Mona Bakery bakes cheese cake which cost $10 per box. The fixed cost to run the bakery is $6030 per week. If the Mona sells the cheese cake for $40 each box,
   (i) how many boxes of cheese cake must be baked weekly to reach the break-even point? [4 marks]
   (ii) what is the break-even revenue? [2 marks]

c) James wishes to produce two types of benches: Type A and Type B. Each Type A bench will result in a profit of $1000, and each Type B bench will result in a profit of $1200. To manufacture a Type A bench requires 2 hours on Machine X and 1 hour on Machine Y. A Type B bench requires 1 hour on Machine X and 3 hours on Machine Y. There are 180 hours available on Machine X and 300 hours available on Machine Y. James would like to maximize the profit earned.
   (i) Formulate a linear programming (LP) model for this problem. [4 marks]
   (ii) Solve the LP problem graphically and state the optimal solution. [9 marks]
   (iii) How many benches of each type should James make to maximize its profit? [1 mark]

Question 2 (25 marks)

a) A financial cooperation has $750,000 to lend for 7 months. It can lend it to a construction company at a simple interest rate of 12%, or it can lend it to a businessman that will pay 12% compounded monthly. If the financial cooperation wants to maximize its interest earned, who should receive the loan (construction company or businessman) and what is the additional interest earned? [8 marks]

b) Muthu knows that he will need to buy a new motorbike in 3 years. The motorbike will cost RM15,000 by then. How much should he invest now at 12%, compounded quarterly, so that he will have enough to buy a new motorbike? [4 marks]

c) After putting $10,000 as a down payment on a unit of apartment, Sofia began paying $2500 quarterly for nine years. Given an interest rate of 7.75% compounded quarterly, how much would the apartment cost if she had paid for it in cash? [7 marks]

d) If $300,000 is to be saved over 25 years, how much should be deposited monthly if the investment earns 8% interest compounded monthly? [6 marks]

Continued...

Question 3 (25 marks)

a) Find the derivatives of the following function:

(i) $y = 3x^5 - \frac{1}{4x^2} + 9\sqrt{x}$ [3 marks]

(ii) $y = (x^3 + 3)^4(2x^2 - 5)$ [4 marks]

(iii) $y = 6e^{2x+3} - ln\,(3x + 4)$ [4 marks]

b) Makcik Kiah sells a curry puff for \$3.50 each. The daily cost function is estimated to be $C(x)$ ringgit, where x is the number of curry puffs sold in a day and the function is given below:

$$C(x) = 0.0006x^3 - 0.03x^2 + 2x + 20$$

(i) Find the revenue function, $R(x)$ and the profit function, $P(x)$ for the above situation. [4 marks]

(ii) Find the number of curry puffs, x, sold that will maximize Makcik Kiah's daily profit. [4 marks]

(iii) Compute average cost and marginal average cost. [4 marks]

(iv) Find the marginal average cost when the x is 100 and interpret the result. [2 marks]

Question 4 (25 marks)

a) For the function $f(x, y) = x^2 + 3xy + y^2 - 10x - 20y + 12$

(i) Find the critical point(s). [5 marks]

(ii) Then use the second derivative test to classify the nature of the critical point(s). [3 marks]

b) Solve the following integral:

(i) $\int_{-1}^{2}(5x^3 + 2x^2 - 13)\ dx$ [5 marks]

(ii) $\int x^2\sqrt{2x^3 - 5}dx$ [5 marks]

c) A manufacturer's marginal revenue function is

$$\frac{dR}{dq} = 0.006q^2 - 0.3q + 20$$

If R is in dollar, determine the revenue involved to increase production from 50 to 100 units. [7 marks]

End of question

Summary of Principal Formulas and Terms

Simple Interest

(i) Interest, $I = Prt$ (P = principal, r = interest rate, t = number of years)

(ii) Accumulated amount, $A = P(1 + rt)$

Compound Interest

(i) Accumulated amount, $A = P(1+i)^n$, where $i = \frac{r}{m}$, and $n = mt$

(m = number of conversion periods per year)

(ii) Present value for compound interest, $P = A(1+i)^{-n}$

Effective Rate of Interest

$$r_{eff} = \left[1 + \frac{r}{m}\right]^n - 1$$

Future Value of an Annuity

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$ (S = future value of ordinary annuity of n payments of R dollars periodic payment)

Present Value of an Annuity

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$ (P = present value of ordinary annuity of n payments of R dollars periodic payment)

Amortization Formula

$$R = \frac{Pi}{1-(1+i)^{-n}}$$ (R = periodic payment on a loan of P dollars to be amortized over n periods)

Sinking Fund Formula

$$R = \frac{Si}{(1+i)^n - 1}$$ (R = periodic payment required to accumulate S dollars over n periods)

Basic Rules of Differentiation

a) Product rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

b) Quotient rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

c) Chain rule: $g[f(x)] = g'[f(x)]f'(x)$

d) General Power rule: $\frac{d}{dx}\left[f(x)^n\right] = nf(x)^{n-1} f'(x)$

e) Logarithmic function: $\frac{d}{dx}(\ln u) = \frac{1}{u}\left(\frac{du}{dx}\right)$

f) Exponential function: $\frac{d}{dx}\left(e^u\right) = e^u \frac{du}{dx}$

Basic Rules of Integration

a) Logarithmic function: $\int \frac{1}{u} du = \ln u + C$

b) Exponential function: $\int e^u du = e^u + C$

Determining Relative Extrema

$$D(x, y) = f_{xx} f_{yy} - (f_{xy})^2$$

If $D > 0$ and $f_{xx} > 0$, relative minimum point occurs at (x, y)
If $D > 0$ and $f_{xx} < 0$, relative maximum point occurs at (x, y)
If $D < 0$, (x, y) is neither maximum nor minimum point
If $D = 0$, the test is inconclusive

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